EVAPORATION TIME DURING DRYING OF A DROP OF SOLUTION OF ENTOBACTERIN BACTERIAL PREPARATION

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An analytical solution of the problem is obtained which allows one to determine the duration of the process of drying a drop when the moisture content in the drop varies linearly and exponentially. The results are compared with experimental data.

The majority of the published reports consider the evaporation of a drop without allowance for the variation in its moisture content in the course of dehydration. We have attempted to determine the temperature field of a drop of solution and the evaporation time with the moisture content being allowed for in a boundary condition. With low-temperature drying the greater part of the moisture is removed in the period of a constant drying rate.

The evaporation time of a single drop of solution — the lifetime of the drop — has great theoretical and practical importance. In a sufficiently general form the problem of the drying of a spherical drop in the presence of a moving phase-transition surface can be formulated as follows:

$$c_1\rho_1\frac{\partial t_1}{\partial\tau}=\lambda_1\left(\frac{\partial^2 t_1}{\partial r^2}+\frac{2}{r}\cdot\frac{\partial t_1}{\partial r}\right),\quad 0\leqslant r\leqslant\varepsilon(\tau),\ \tau>0, \tag{1}$$

$$c_2\rho_2\frac{\partial t_2}{\partial\tau}=\lambda_2\left(\frac{\partial^2 t_2}{\partial r^2}+\frac{2}{r}\cdot\frac{\partial t_2}{\partial r}\right),\quad \varepsilon(\tau)\leqslant r\leqslant R,\ \tau>0, \tag{2}$$

$$t_1(r,\ 0)=t_0, \tag{3}$$

$$-\lambda_2\frac{\partial t_2(R,\ \tau)}{\partial r}+\alpha\left[t_{\mathrm{m}}-t_2(R,\ \tau)\right]=0, \tag{4}$$

$$\frac{\partial t_1(0,\ \tau)}{\partial r}=0,\quad t_1(0,\ \tau)\neq\infty. \tag{5}$$

The conditions at the moving evaporation (boiling) surface, i.e., at $r=\varepsilon(\tau)$:

$$t_1(\varepsilon,\ \tau)=t_2(\varepsilon,\ \tau)=t_{\mathrm{evap}}, \tag{6}$$

$$\lambda_2\frac{\partial t_2(\varepsilon,\ \tau)}{\partial r}-\lambda_1\frac{\partial t_1(\varepsilon,\ \tau)}{\partial r}=-L\rho_1 u(\tau)\frac{d\varepsilon}{d\tau}. \tag{7}$$

The condition (7) expresses the fact that the heat flux entering through the surface $r=\varepsilon$ of the drop is partly expended on evaporation of the liquid at the moving surface and is partly drawn into the drop and produces a change in the temperature of the wet region of the drop. During the drying the moisture content of the drop and, consequently, the concentration of the dissolved substance vary continuously. This property of the process is taken into account by the introduction of the function $u(\tau)$, which determines the average moisture content of the wet region of the drop at any time, into the boundary condition (7) at the moving surface. This function carries primary information concerning the nature of the process and its specific properties.

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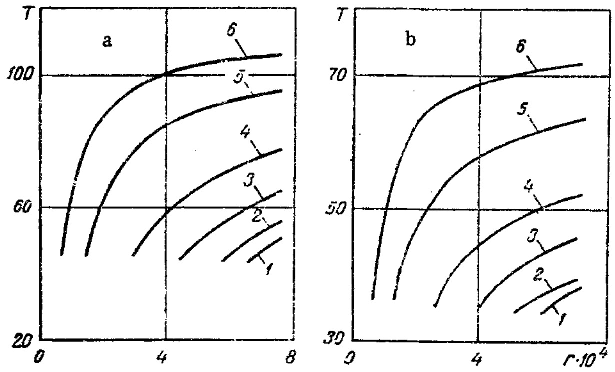

Fig. 1. Temperature field of a drop of solution during the movement of the evaporation boundary: a) with the temperature of the medium $t_m = 120°C$: 1) $\tau_1 = 98$ sec, $\varepsilon = 0.9R$; 2) $\tau_2 = 137$ sec, $\varepsilon = 0.8R$; 3) $\tau_3 = 161$ sec, $\varepsilon = 0.6R$; 4) $\tau_4 = 167$ sec, $\varepsilon = 0.4R$; 5) $\tau_5 = 169$ sec, $\varepsilon = 0.2R$; 6) $\tau_6 = 170$ sec, $\varepsilon = 0.1R$; b) with the temperature of the medium $t_m = 82°C$: 1) $\tau_1 = 165$ sec, $\varepsilon = 0.9R$; 2) $\tau_2 = 225$ sec, $\varepsilon = 0.8R$; 3) $\tau_3 = 262$ sec, $\varepsilon = 0.6R$; 4) $\tau_4 = 267$ sec, $\varepsilon = 0.4$ R; 5) $\tau_5 = 269$ sec, $\varepsilon = 0.2$ R; 6) $\tau_6 = 270$ sec, $\varepsilon = 0.1R$. T, °C; r, m.

The drying of the drop of solution proceeds with Fourier numbers Fo of (1-40) and relatively small Piot numbers Bi (0.3-1.0). Let us find an approximate solution of the problem (1)-(7) in the case when the temperature distribution in the wet region of the drop is uniform and constant, i.e., $t_1(r, \tau) = t_0$.

The temperature within the drop can be taken as constant and equal to the wet-bulb temperature in the first period of a constant drying rate, while the temperature of the dried part of the drop increases during the drying.

The temperature distribution in the dry region of the drop is determined by the function

$$t_2(r, \tau) = t_{evap} + \frac{t_m - t_{evap}}{\frac{1}{R} - \frac{1}{\varepsilon} - \frac{\lambda_2}{\alpha R^2}} \left(\frac{1}{r} - \frac{1}{\varepsilon}\right), \quad \varepsilon(\tau) \leqslant r \leqslant R. \tag{8}$$

Equation (8) satisfies the conditions (4) and (6). By substituting the value of t_2 from (8) into the boundary condition (7) and assuming the constancy of the temperature in the wet region of the drop, we find

$$\frac{[t_m - t_{evap}]\, d\tau}{u(\tau)} = -\frac{L\rho_1}{\lambda_2}\left(A + \frac{1}{\varepsilon}\right)\varepsilon^2 d\varepsilon, \tag{9}$$

where $A = \lambda_2/\alpha R^2 - 1/R$.

Equation (9) determines the law of motion of the phase-transition surface, i.e., the law of decrease of the wet zone of the particle of solution during the drying, and is easily integrated if the function $u(\tau)$ is known.

One can assume that in the period of a constant drying rate the moisture content varies linearly with time while the evaporation temperature is constant.

Therefore, assuming that

$$u(\tau) = u_i - K\tau, \tag{10}$$

$$t_{evap} = \text{const}, \tag{11}$$

we obtain from Eq. (9)

$$\frac{d(u_i - K\tau)}{u_i - K\tau} = KM\left(A + \frac{1}{\varepsilon}\right)\varepsilon^2 d\varepsilon, \tag{12}$$

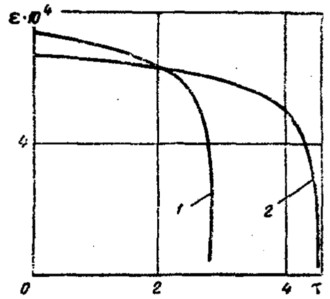

Fig. 2. Curves of movement of evaporation boundary in a drop of solution: 1) with t_m = 120°C; 2) with t_m = 82°C; ε, m; τ, min.

where

$$M = \frac{L\rho_1}{\lambda_2 (t_m - t_{evap})}. \tag{13}$$

Let us integrate Eq. (12), determining the integration constant from the condition that ε(0) = R at τ = 0:

$$\tau = \frac{u_i}{K}\left[1 - \exp\left\{-\frac{KM}{6}[2A(R^3-\varepsilon^3) + 3(R^2-\varepsilon^2)]\right\}\right]. \tag{14}$$

Under the conditions which we are considering the drying time of a drop of solution can be determined from Eq. (14) if one considers that $\varepsilon(\tau_{end}) = 0$ when $\tau = \tau_{end}$, i.e.,

$$\tau_{end} = \frac{u_i}{K}\left[1 - \exp\left\{-\frac{KM}{6}[2AR^3 + 3R^2]\right\}\right]. \tag{15}$$

Experimental studies which have been performed [3] on the kinetics of the evaporation of drops of culture liquid of a bacterial preparation of entobacterin with different concentrations and temperatures of the heat-transfer agent provided us the opportunity to determine the drying rate coefficient K. It can be represented in the form of the dependence

$$K = 2.3\cdot 10^{-7} t^{2.1} \cdot (u_i + 2.7). \tag{16}$$

The error is ±10%. The dependence is valid within the limits of t = 80-150°C and u_i = 7.2-14.4 kg/kg.

t high evaporation temperatures the first period of the process of drying a drop of solution is very brief or altogether absent and one can assume that the drying curve varies with time according to the following law:

$$u(\tau) = u_i e^{-k\tau}, \tag{17}$$

so that (9) takes the form

$$\frac{t_m - t_{evap}}{u_i e^{-k\tau}} \cdot d\tau = -\frac{L\rho_1}{\lambda_2}\left(A + \frac{1}{\varepsilon}\right)\varepsilon^2 d\varepsilon. \tag{18}$$

After integration of Eq. (18) and the determination of the integration constant, we obtain

$$\tau = \frac{1}{k}\ln\left\{1 - u_i kM\left[\frac{A}{3}(R^3-\varepsilon^3) + \frac{1}{2}(R^2-\varepsilon^2)\right]\right\}. \tag{19}$$

If $\tau = \tau_{end}$ then $\varepsilon(\tau_{end}) = 0$ and

$$\tau_{end} = \frac{1}{k}\ln\left[1 - \frac{u_i kM}{6}(2AR^3 + 3R^2)\right]. \tag{20}$$

Using the average thermophysical characteristics obtained for the dry bacterial preparation and for the entobacterin culture liquid [4], we found the evaporation time for drops with temperatures of 82 and 120°C, $\bar{u}_i$ = 7.25 kg/kg, and diameters of $1.3\cdot 10^{-3}$ and $1.5\cdot 10^{-3}$ m. The average heat-exchange coefficient was calculated from the well-known generalized equation of [5] with allowance for the correction $(u/u_{cr})^n$ and the Rebinder number which we introduced [3]:

$$\overline{Nu} = (2 + 0.51\,\overline{Re}^{0.52}\,\overline{Pr}^{0.33})\left(\frac{u}{u_{cr}}\right)^{0.57}(1 + \overline{Rb}). \tag{21}$$

Calculations made through Eqs. (15) and (16) showed that the drying time for a drop of solution is equal to 4.5 min at t = 82°C and 2.8 min at t = 120°C, which is 15% less than the time determined from the drying curves obtained experimentally.

The temperature field of the dry particle, which varies as the evaporation surface moves, is presented in Fig. 1a, b, while the movement of the evaporation boundary is shown as a function of the evaporation time in Fig. 2.

NOTATION

t_1, temperature of wet part of drop; t_2, temperature of dry part of drop; r, variable radius of drop; R, initial radius of drop; τ, time; α, heat-exchange coefficient; t_m, temperature of medium; L, latent heat of vaporization; K, drying coefficient; u, moisture content; λ_1 and λ_2, thermal-conductivity coefficient of liquid and of dry material; ρ_1, ρ_2, density of solution and of dry material.

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